A Visual Proof of Amortised-linear Resizable Arrays

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ABSTRACT

We demonstrate visually why doubling capacity is the better strategy when resizing arrays. The visual proof makes simple amortised analysis more accessible to a CS2 audience.

Categories and Subject Descriptors

K.3.2 [**Computers and Education**]: Computer and Information Science Education—*Computer science education*

General Terms

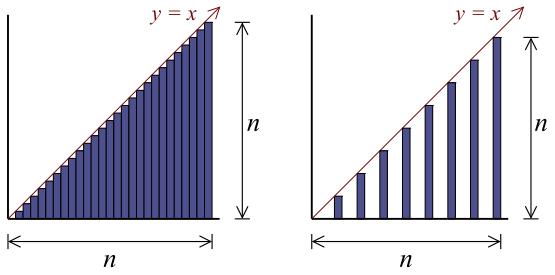
Algorithms

Keywords

Amortised analysis, visual proof

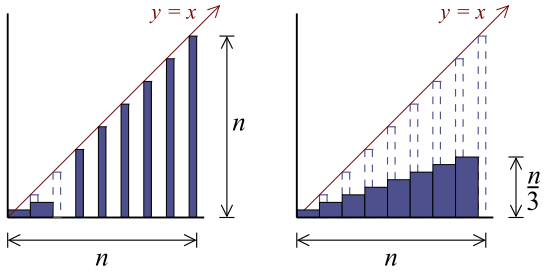
1. THE PROOF

CS2 students are usually quick to understand that increasing an array's size by only one, necessitating a resize with every single addition, is too expensive. Their usual first suggestion, however, is to increase the size by a constant amount. We can diagram the amount of work required for each strategy as follows (here with an interval of three):



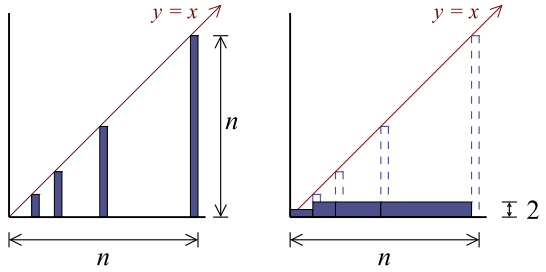
We draw the amount of copying work at time i as a width-1 vertical bar whose height corresponds to the number of operations. On the left, the total amount of work can be computed geometrically: the area of the triangle with base n and height n is $\frac{n^2}{2}$. The geometric analysis of the diagram on the right is slightly less obvious.

To make it clearer, we perform a diagram transformation that makes each vertical bar "tip over" or "melt" into the open space to its left, creating a wider, shorter rectangle equal in area to the original:



We now have a jagged-edged triangle with an area, or total copying work load, of (about) $\frac{1}{2} \cdot n \cdot \frac{n}{3}$ or $\frac{n^2}{6}$.

The stage is now set for a visual proof of the amortised-linear cost of resizing at increasing intervals. We now start with a size of three and double whenever full; and now when we "tip over" the rectangles into the spaces to their left, all but the first fall to a height of exactly 2:



The work done fits strictly within a rectangle of area $2n$.

2. CONCLUSIONS

Visual proofs are not new, and the fifth diagram above is similar to one in at least one CS2 textbook.[1] However, we believe that the "tip over" diagram transformation is new, and the visual proof is a compelling one for CS2 students who have never seen amortised analysis before. The transformation is exactly analogous to the well-known accounting technique of saving two credits or "cyber-dollars" per add operation for the eventual copy-and-resize.

3. REFERENCES

[1] M. T. Goodrich and R. Tamassia. *Data structures and algorithms in Java.* 2006.

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