

A Visual Proof of Amortised-linear Resizable Arrays

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Visual Proof of Linear Resizable Arrays

Resizable arrays

- CS2-level concept (e.g. ArrayList, vector)
- Key algorithm:

add (element)

if size = capacity

allocate new array that is larger

copy elements into new array

dispose of old array

update capacity

add element at end of array

increment size

Visual Proof of Linear Resizable Arrays

How much larger?

- Incrementing by one works, but seems wasteful
 - Resize is $O(n)$ for copying
 - So adding n elements is $O(n^2)$
- Adding 3 is better
 - Resize still $O(n)$, but sometimes not required
 - But adding n elements is $O(n^2)$
- Even better. . .

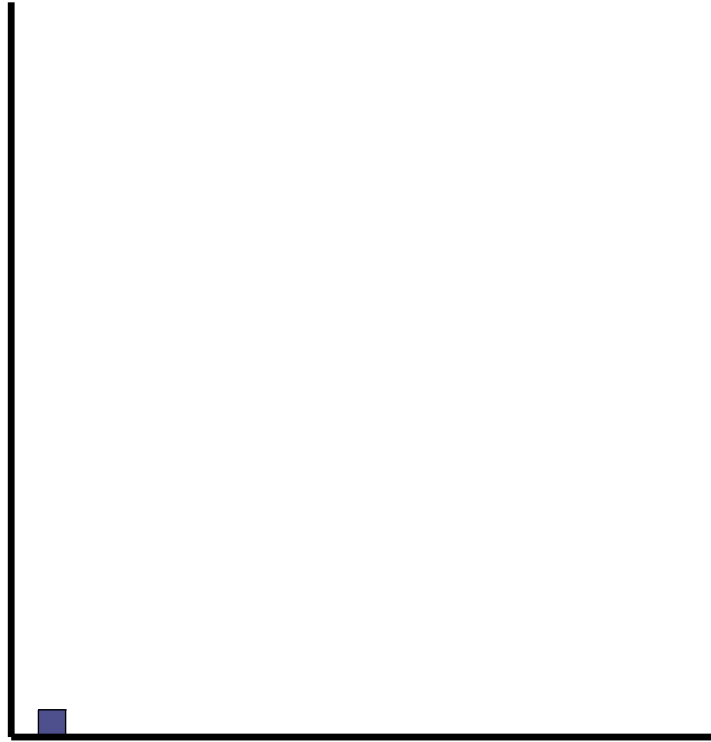
Visual Proof of Linear Resizable Arrays

Double the size each time!

- *Multiplying* the size is best
- Adding n elements is now $O(n)$
- Proof? Amortised analysis. . .

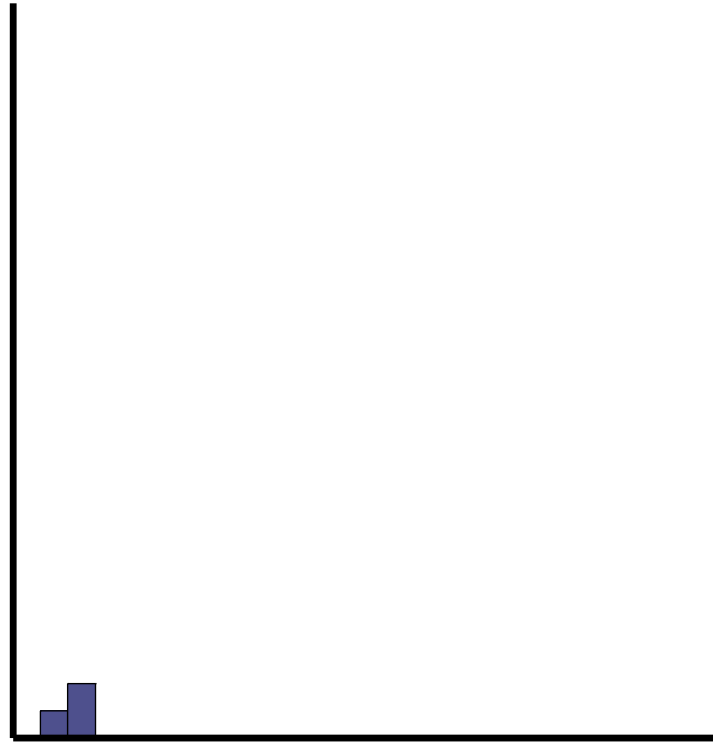
Visual Proof of Linear Resizable Arrays

First, show the problem visually



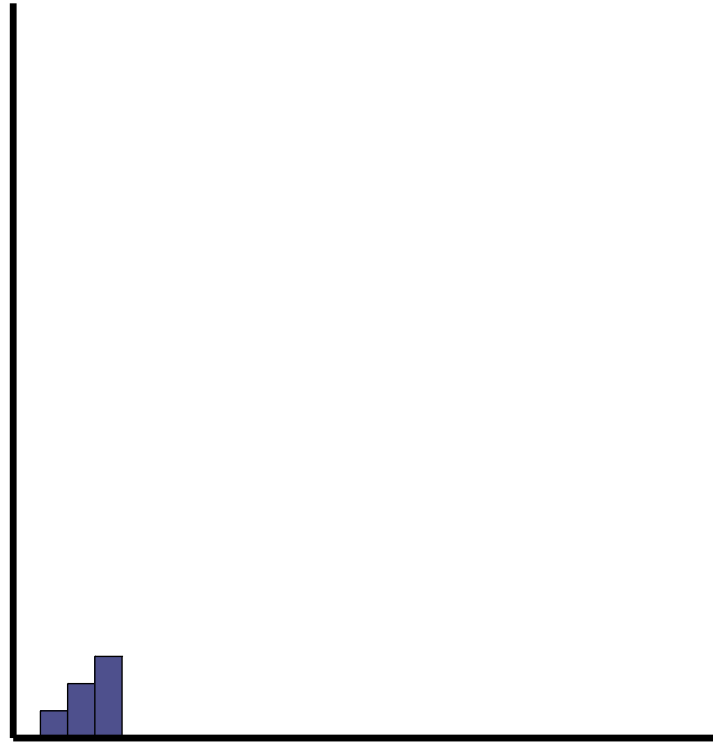
Visual Proof of Linear Resizable Arrays

First, show the problem visually



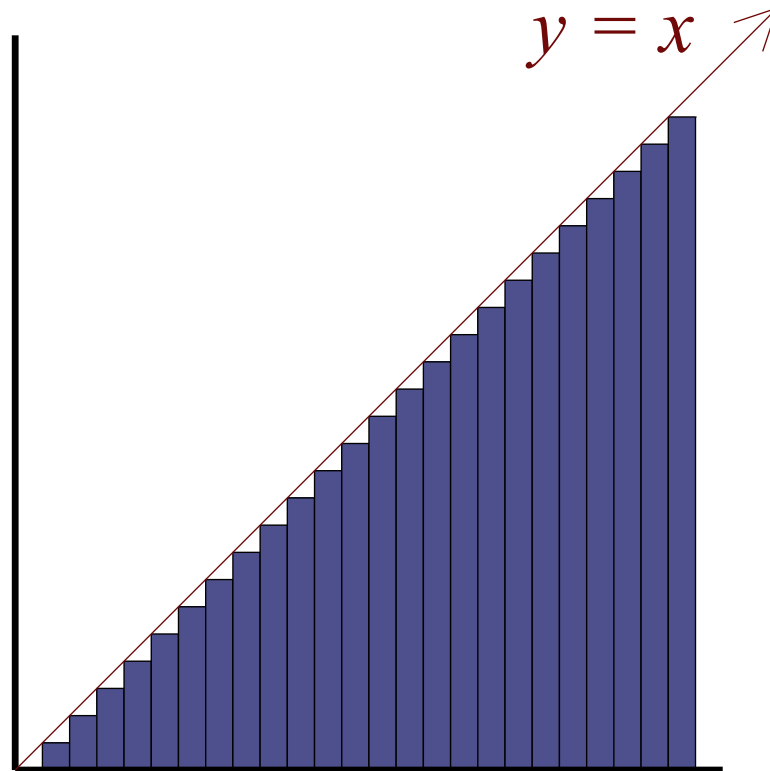
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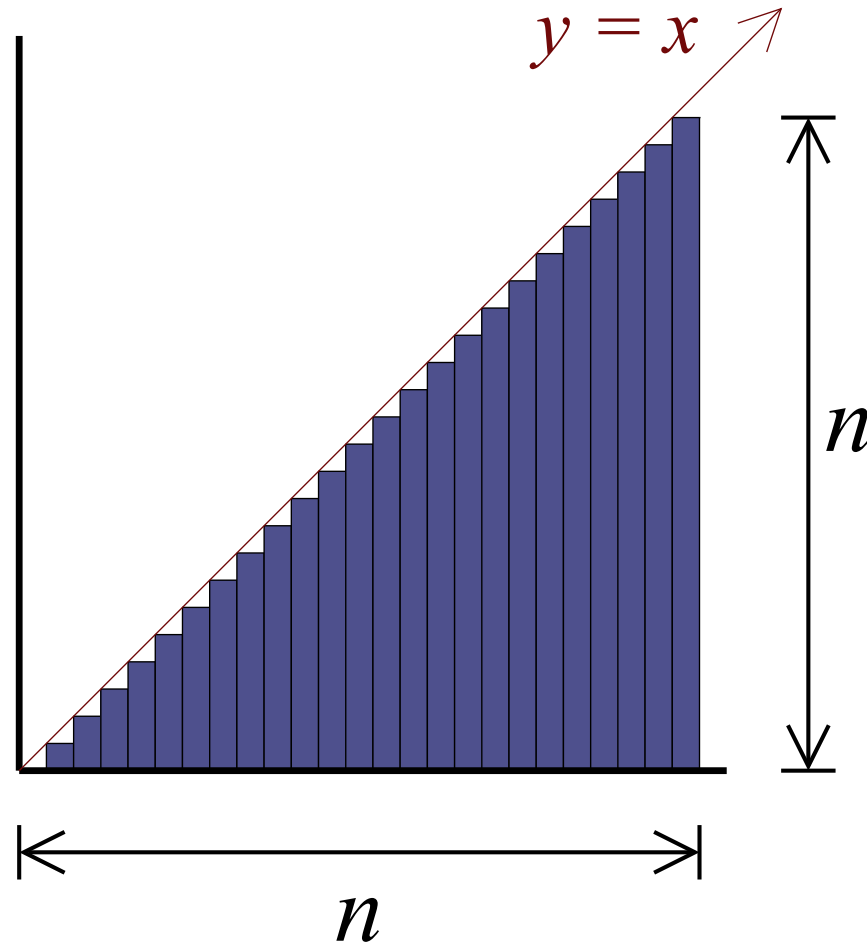
Visual Proof of Linear Resizable Arrays

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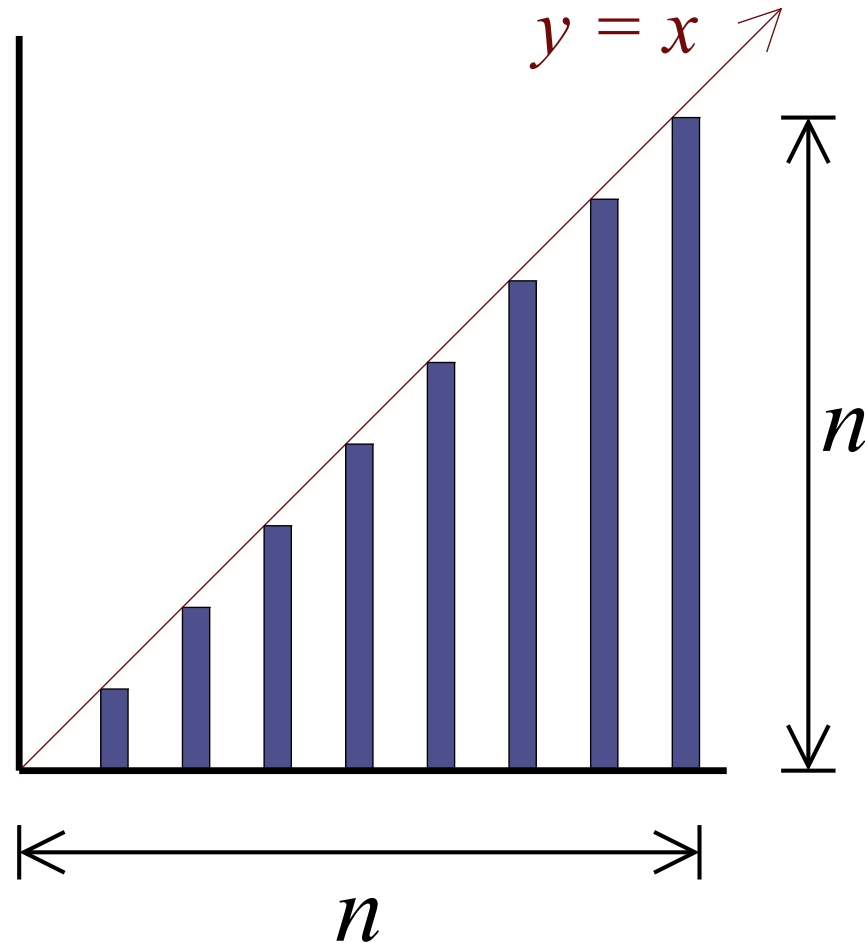
Visual Proof of Linear Resizable Arrays

First, show the problem visually



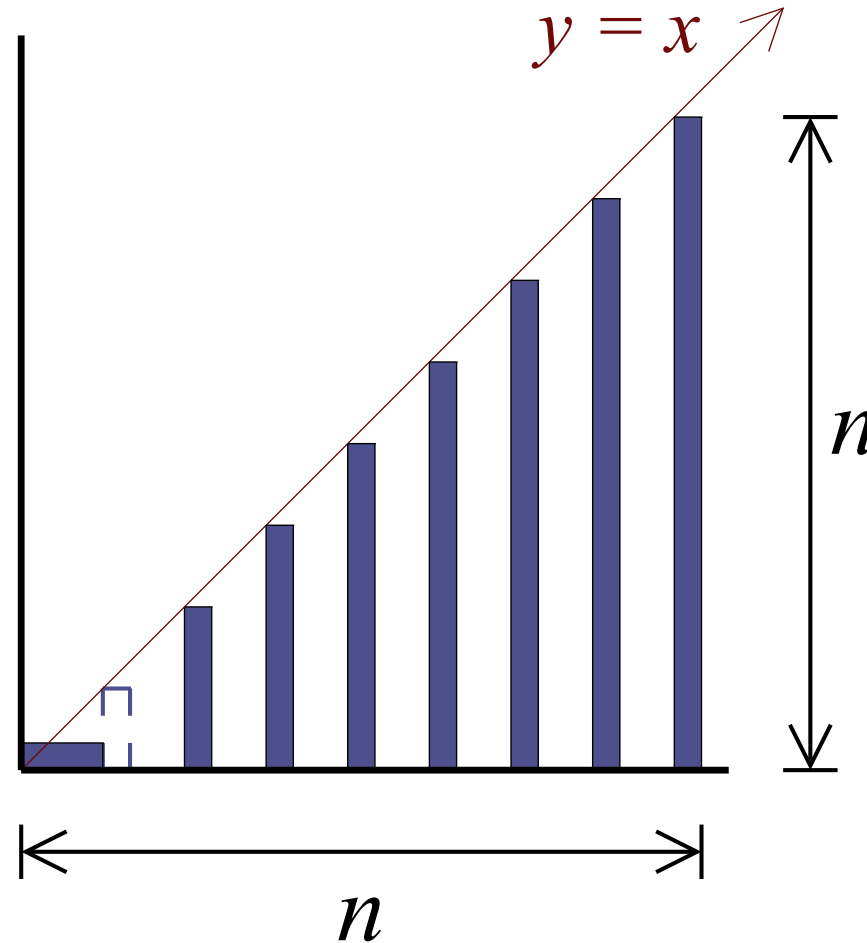
Visual Proof of Linear Resizable Arrays

Increment capacity by 3?



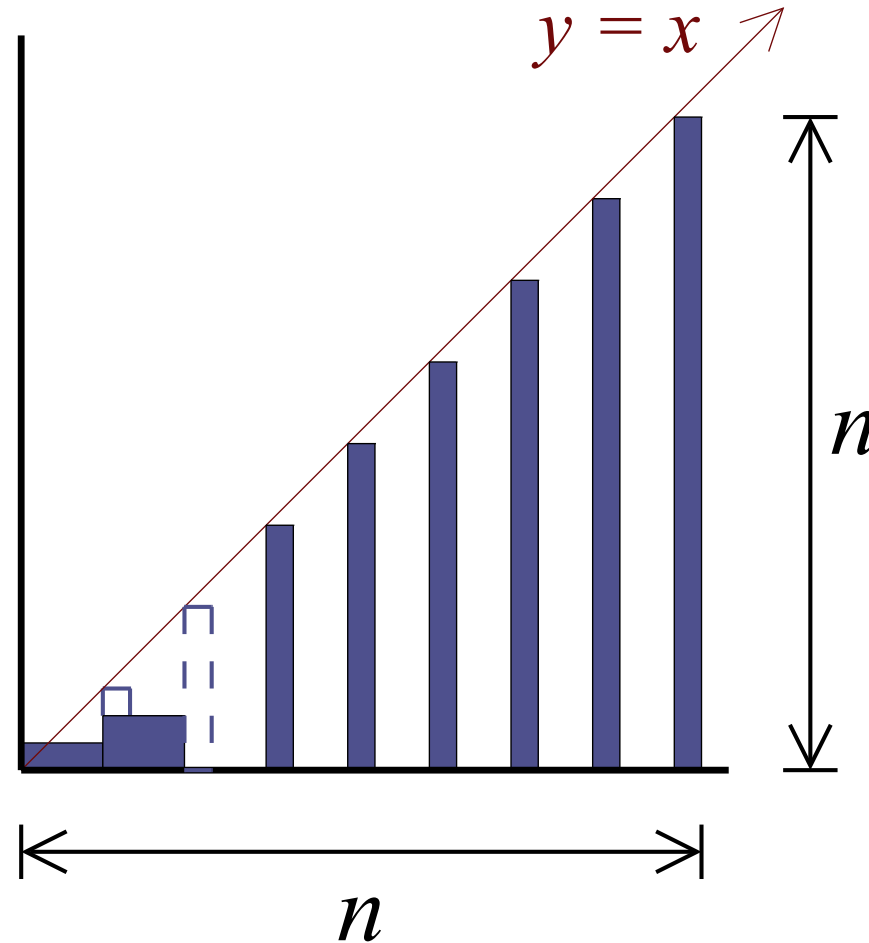
Visual Proof of Linear Resizable Arrays

Form a solid area...



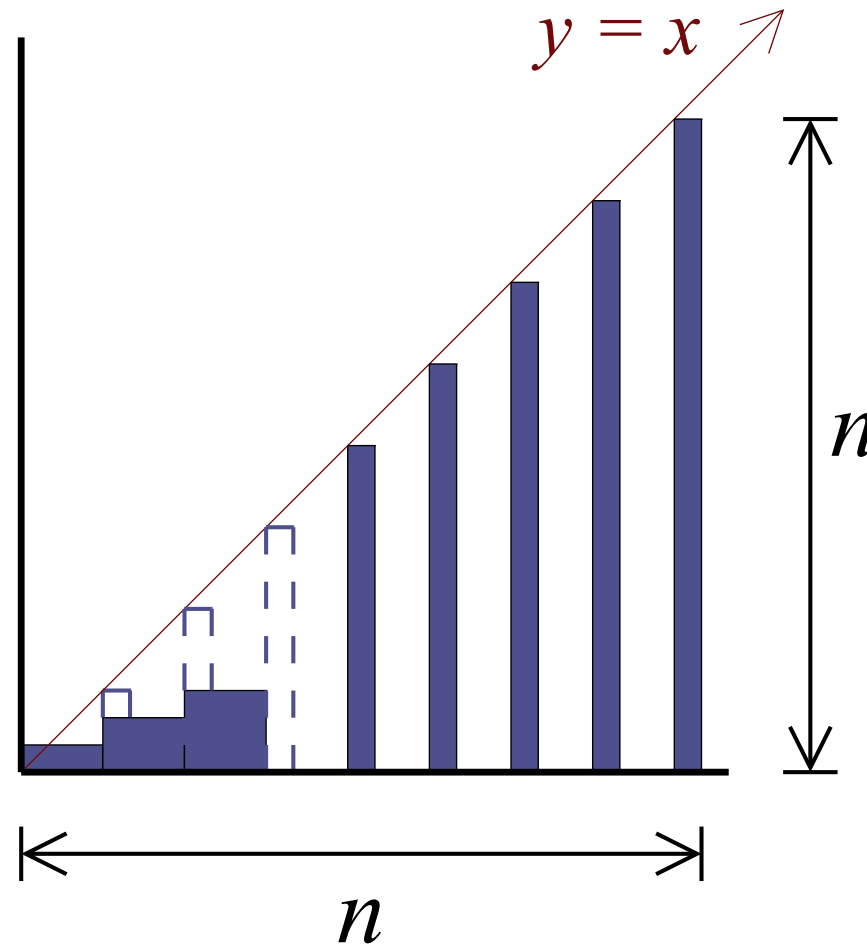
Visual Proof of Linear Resizable Arrays

Form a solid area...



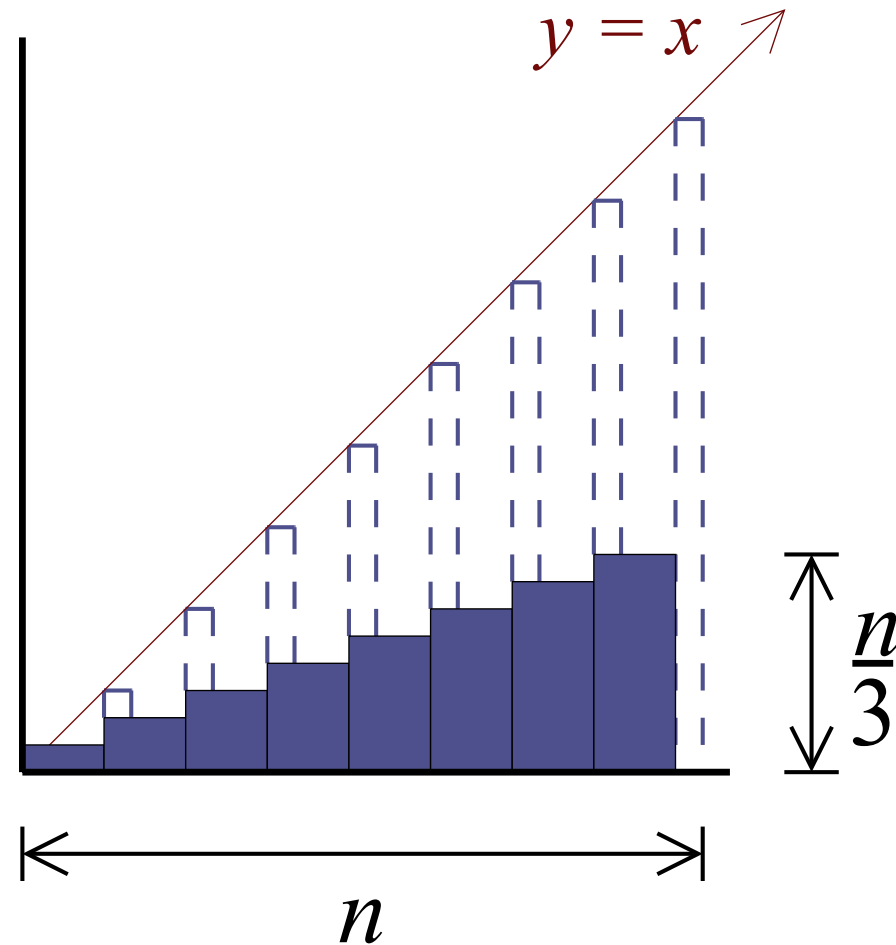
Visual Proof of Linear Resizable Arrays

Form a solid area...



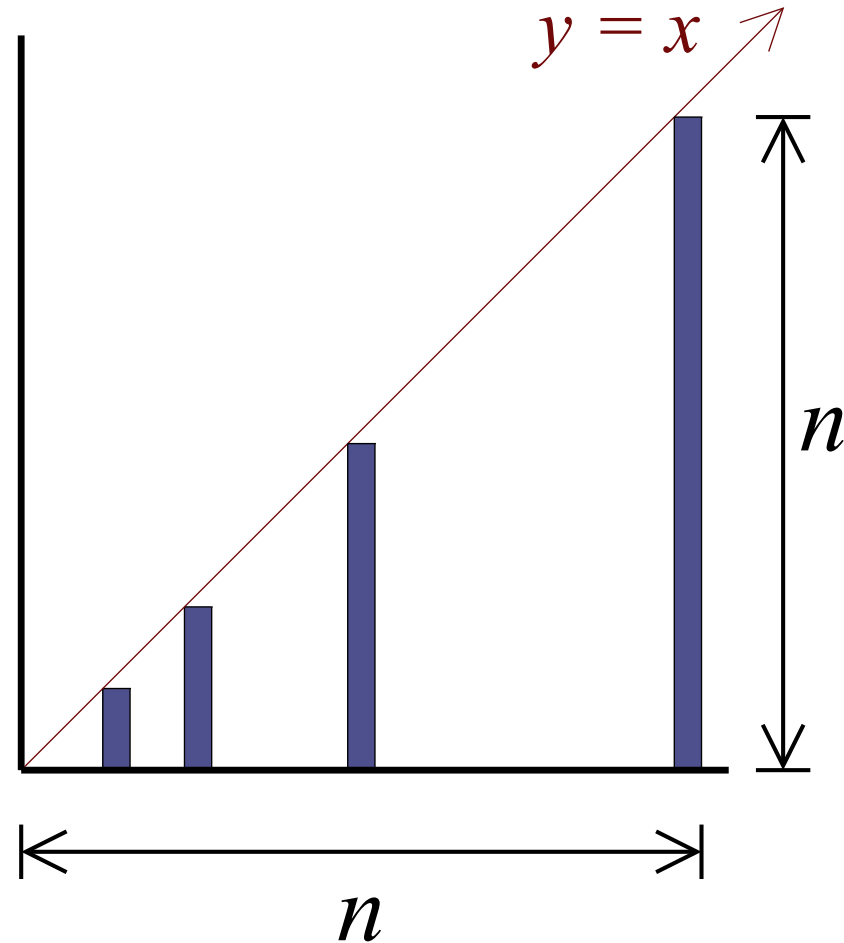
Visual Proof of Linear Resizable Arrays

Form a solid area...



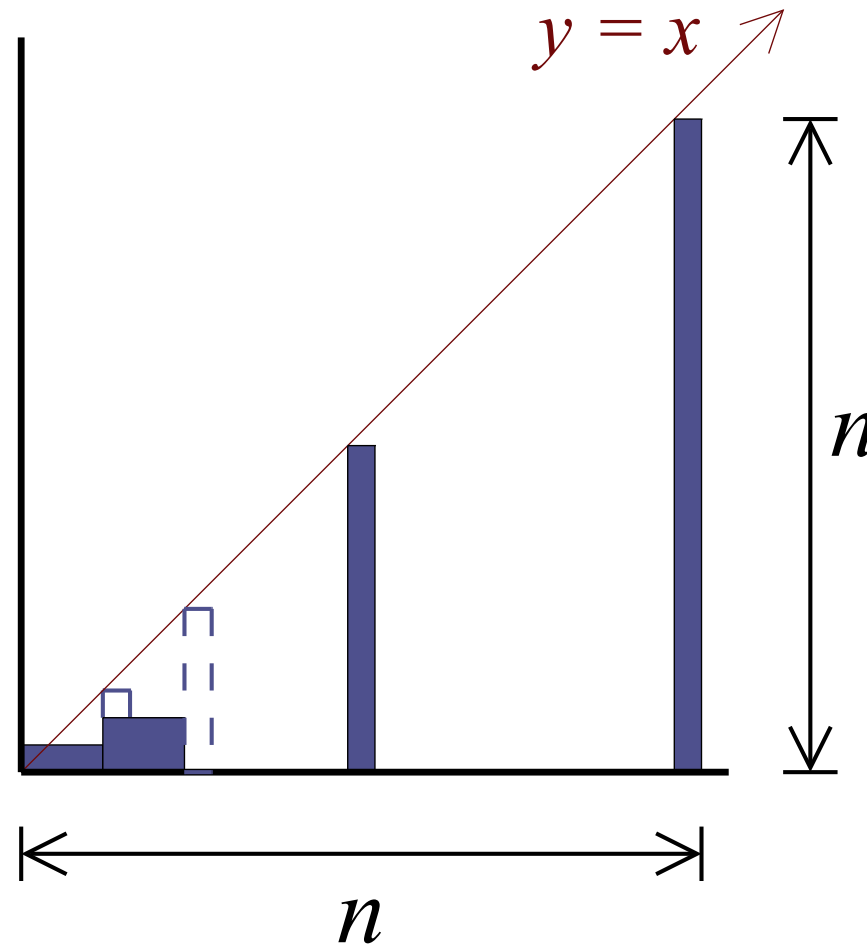
Visual Proof of Linear Resizable Arrays

Double each time



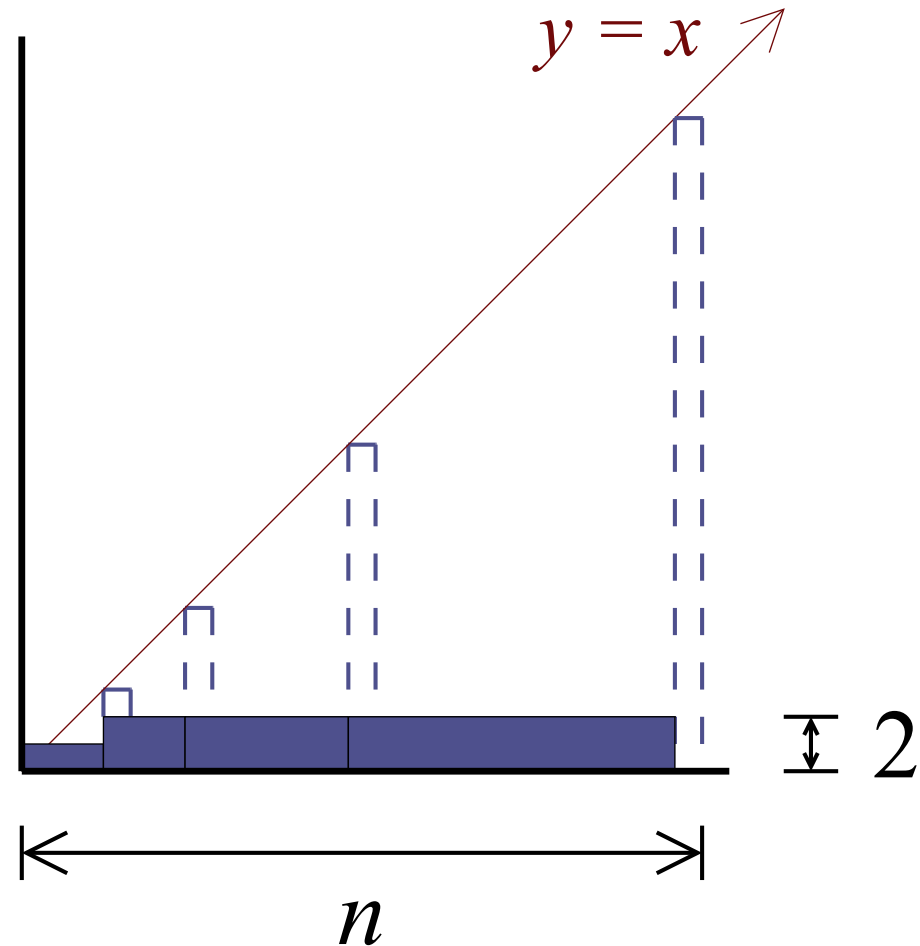
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Starts out the same:



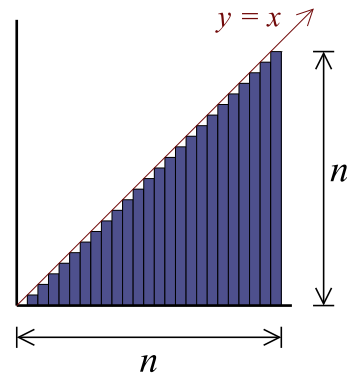
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But now height remains constant

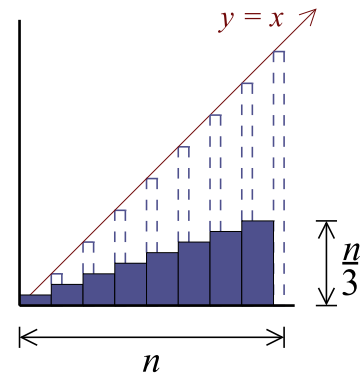


Visual Proof of Linear Resizable Arrays

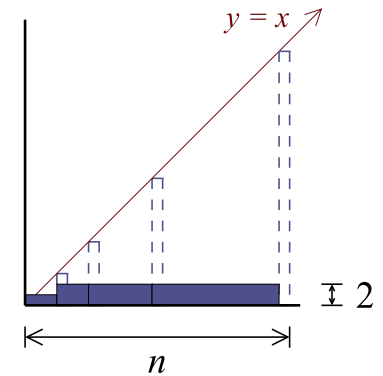
Comparison



Area: $\frac{1}{2} \times n \times n$
Analysis: $O(n^2)$



$\frac{1}{2} \times \frac{n}{3} \times n$
 $O(n^2)$



$2 \times n$
 $O(n)$

Visual Proof of Linear Resizable Arrays

Summary

- General visual proof strategy:
 - Start with 'work done' as bar chart
 - Rearrange pieces to form solid shape
 - Apply area formula
- More convincing to new students
- Accessible in CS2